SUPPLEMENT B TO "TWO-SAMPLE KOLMOGOROV-SMIRNOV TYPE TESTS REVISITED: OLD AND NEW TESTS IN TERMS OF LOCAL LEVELS":

ANIMATED GRAPHICS OF LOCAL LEVELS*

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In this supplement to [1] we illustrate the convergence of two-sided local levels related to two-sample weighted KS tests and minP tests with the help of animated plots. Several plots in the figures of the main manuscript are related to the animated plots in Supplement B. Note that Figures 2-9 (except the right graph in figure 4) in the main manuscript display lower local levels, while all figures in this supplement show two-sided local levels.

The left graph in Figure 5 relates to the last plot in Figure B2, the left graph in Figure 6 relates to the last plot in Figure B3, the left graph in Figure 7 relates to the last plot in Figure B4, the left graph in Figure 9 relates to the last plot in Figure B5 and the right graph in Figure 10 relates to the last plot in Figure B6.

The respective first plot for m = n = 50 in Figures B1, B2 and B3 is related to the corresponding plot of lower local levels in Figure 2. The first plot for m = n = 50 in B4 is related to left tail lower local levels for $\nu = 0.75$ in Figure 2. Finally, the first plot for m = n = 50 in Figure B5 is related to the lower local levels plot in Figure 8.

All interactive animations can be controlled by the button bar at the bottom of the figures.

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FIG B1. Two-sided local levels (black points) of the original two-sided KS tests at level $\alpha = 0.05$ together with corresponding asymptotic local levels (red curve) based on the asymptotic KS critical value $b_{\alpha} = 1.3581$.

FIG B2. Two-sided local levels (black points) of weighted KS tests with $\nu = 0.25$ at level $\alpha = 0.05$ together with corresponding asymptotic local levels (red curve) based on the asymptotic critical value b = 2.002 (simulated by 10^6 repetitions).

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FIG B3. Two-sided local levels (black points) of HC tests (i.e., weighted KS tests with $\nu = 0.5$) at level $\alpha = 0.05$ together with corresponding asymptotic local levels (red line).

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FIG B4. The first 25 two-sided local levels (black points) of weighted KS tests with $\nu = 0.75$ at level $\alpha = 0.05$ together with corresponding asymptotic local levels (red points) based on the asymptotic critical value b = 10.46635 (simulated by 10^6 repetitions).

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FIG B5. Two-sided local levels (black points) of minP GOF tests at level $\alpha = 0.05$ together with corresponding asymptotic local levels (red line).

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FIG B6. Two-sided local levels α_s , $s \in I_{m,n}$, (black points) of level α GOF tests based on the sum of original KS and minP local levels for $\alpha = 0.05$. Thereby, α_s , $s \in I_{m,n}$, are as large as possible fulfilling $\alpha_s \leq \alpha_s^{KS} + \alpha_s^{minP}$, $s \in I_{m,n}$, where $\alpha_s^{KS} = 2\Phi(-b/\sqrt{s/(m+n)(1-s/(m+n))})$ with an approximate asymptotic two-sided KS critical value $b = \sqrt{-\log(\kappa/2)/2}$ and $\alpha_s^{minP} = -\log(1-\kappa)/(2\log(m)\log(\log(m)))$. Note that κ is chosen such that the combined tests are level α tests. The upper bound $\alpha_s^{KS} + \alpha_s^{minP}$, $s \in I_{m,n}$, is given by the blue curve, while the asymptotic local levels, given by the red curve, are equal to the upper bound for $\kappa = \alpha/2$.

References.

[1] FINNER, H. and GONTSCHARUK, V. (2017). Two-sample Kolmogorov-Smirnov type tests revisited: Old and new tests in terms of local levels. *Ann. Statist.*, accepted for publication.

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